HOW TEACHERS' KNOWLEDGE IS INFLUENCED BY A PROFESSIONAL LEARNING INTERVENTION IN ESTIMATION

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Abstract

This research aims to study the change in the performance of 11 in-service Greek primary school teachers, following an educational intervention in estimation. The factors focused on this survey, in order to measure the change of teacher performance, related to their cognitive characteristics. These factors are: content knowledge (CK), pedagogical content knowledge (PCK) based on questionnaire findings, PCK based on observation of the teaching process, estimation strategy use, and student performance in estimation problems. Results showed that student performance seems to be mainly influenced by the teaching method in classroom. Another correlation found is between CK and estimation strategy use. Also noteworthy is the fact that the results of PCK are different and not correlated, when the inquiry is carried out with questionnaires, compared to it being carried out by observing the act of teaching. This raises questions regarding the validity of PCK inquiry through the use of questionnaires.

Keywords: estimation, primary school teachers, in-service teachers, professional learning intervention, teacher learning community.

1. Introduction

Estimation is a very important process in the lives of both children and adults (Star, Rittle-Johnson, Lynch, & Perova, 2009). The term "estimation" may refer to a numerical or measure approach or even to a hypothetical answer given when solving a problem (Rubenstein, 1983). Estimation can be used in everyday life far more often than any other quantification process. We come across four types of estimation on a daily basis: (a) computational estimation, (b) measurement estimation, (c) numerosity estimation, and (d) number line estimation (Koyama 1993; Siegler & Booth 2004; Sarama & Clements, 2009).

More specifically, (a) number line estimation refers to estimation of a number's position in a line, or estimating the number representing a point in a line (Lemonidis, 2013), (b) measurement estimation refers to continuous values, such as length, height, volume, time, weight, etc., but does not refer to discrete sizes like numerosity estimation (Lemonidis, 2013), (c) quantity estimation, or numerosity estimation, refers to the approximate calculation of a discrete quantity. When estimating the number of objects, the question usually asked is how many. For example, how many children are in a room or how many candies are in a jar? (Sarama & Clements, 2009). Lastly, (d) computational estimation refers to finding an approximate answer to an oral or written arithmetic problem without the use of a calculator or other supporting tools. Such an answer can be given by combining mental calculations, number sensing, technical arithmetic skills (digit position value, rounding etc.) which can be rapidly applied to find an adequate answer to everyday life problems, but also judge the reasonableness of a result (Koyama, 1993; Lemonidis, 2002; Mindenhall, Hackling & Swan, 2009; Noordin, Razak & Ali, 2012).

The majority of surveys focus on the most commonly used type of estimation, computational estimation (e.g. Tsao, & Pan, 2013). The surveys carried out so far are studying teachers' views on computational estimation (Alajmi, 2009), as well as teachers' knowledge regarding computational estimation strategies (e.g. Lemonidis & Kaimakami, 2013; Tsao & Pan, 2013). In addition, learning and teaching trajectories have been designed for all types of estimation (Van den Heuvel-Panhuizen, 2001), however, no organized and integrated educational intervention regarding teachers has been implemented based on these trajectories, according to the bibliography of this study. Thus, an effort has been made in this paper to apply a professional learning intervention, which is based on the Common Core State Standards of Mathematics of America (CCSSM, 2011) and adapted to the Greek mathematical educational reality. It concerns in-service teachers who practice all types of estimation, and all estimation strategies, with the ultimate goal of improving their content knowledge and their pedagogical content knowledge, as well as improving their 3rd grade student performance in estimation. Furthermore, this professional learning intervention in estimates is an attempt to investigate the change of teacher behavior. The factors being investigated concern the cognitive characteristics of teachers. This category of factors includes: content knowledge, pedagogical content knowledge based on findings from questionnaires, pedagogical content knowledge based on observation of the teaching process, estimation strategy use, and student performance in estimation problems.

2. Suggested learning and teaching trajectory regarding estimation for 3rd Grade students

A Learning and Teaching Trajectory reflects the overall view of the student learning experience in a particular subject area of the math curriculum (e.g. estimation) and aims at transparency and accessibility in their educational course. It should be noted that the sub-trajectories of each trajectory, the sub-trajectories of different trajectories, and even the different trajectories themselves are associated, they intersect, and they often consolidate, actions that cannot always be easily detected, and clearly described. In mathematics education there are various definitions for learning trajectories. Van den Heuvel-Panhuizen (2001) states that the term of learning and teaching trajectory consists of three interconnected concepts: (a) learning trajectory: giving a general overview of the learners' learning process; (b) teaching trajectory: consisting of teaching indications describing how teaching can be linked more effectively with the learning process, and (c) subject matter outline: indicating which of the basic elements of mathematics should be taught.

As part of this study, a learning and teaching trajectory for estimation for all primary school grades has been created. However, attention was focused on the trajectory of the 3rd Grade, in which, according to the CCSSM (2011), all four types of estimation are included. Specifically, in 3rd Grade, apart from the three types of estimation (number line estimation, numerosity estimation, measurement estimation) which have appeared in previous grades of primary school, computational estimation makes its appearance. Below is a detailed description of the trajectory of all four types of estimation.

More specifically, as regards number line estimation, 3rd Grade students use visualizations and know the relationship between numbers, including the decimal place value, when determined the size and position of a number. In this grade, students extend the knowledge they acquired in the previous two grades in managing numbers up to ten thousand (Sarama & Clements, 2009). In addition, in the same grade, students are required to represent fractional units (e.g. 1/b) to a number line divided into b equal segments, and to recognize that each segment has a size of 1/b (CCSSM, 2011). By acquiring this knowledge, an attempt is made to extend the number line estimations to fractional numbers, while at the same time informally introducing the Special Numbers Strategy (one of the computational estimation strategies).

In terms of numerosity estimation, 3rd graders use perceptual estimation strategies of quantities. In this case they can perceive the size of a quantity by comparing it to the size of another known quantity. For example, they can calculate the height of a child, based on the height of another child known to them. Also, using perceptual strategies, estimation may be drawn from measuring part of a collection which is used as a benchmark (e.g. 5, 10 or 20 items). For example, first they can estimate directly a part of a collection, identifying the number of such parts, and then they can multiply it to obtain the estimate (Sarama & Clements, 2009). More specifically, in 3rd Grade an attempt is made to acquire knowledge of specific perceptual estimation strategies (using a known quantity in an attempt to estimate an unknown quantity) (Sarama & Clements, 2009).

In measurement estimation, 3rd graders also learn to estimate other units apart from length, which they learned in the previous two grades. Specifically, they deal with the estimation of volume, and weight. These estimates include the use of measurement units. More specifically, a main goal of 3rd Grade is to teach how to measure and estimate liquid volumes using standard liter units (l). Additional goals include learning to add, subtract, multiply, and divide in volume related problems (numbers are provided in the same measurement units) (CCSSM, 2011). Furthermore, another target is the measurement and estimation of objects' masses using standard kilogram (kg) and gram (g), as well as addition, subtraction, multiplication, and division in mass problems (numbers are provided in the same measurement units) (CCSSM, 2011). Lastly, a final target is estimating the perimeter of a rectangle (CCSSM, 2011).

Computational estimation first appears in 3rd Grade. Students use estimation when solving problems (along with the four mathematical operations) to judge the reasonableness of a result.

Mental calculations and estimation strategies, including rounding, are used in estimating the reasonableness of answers (CCSSM, 2011). Rounding is learned in 3 phases according to Van den Heuvel-Panhuizen (2001): a) Informal rounding off. b) Informal rounding in additions and subtractions. (c) Informal rounding in multiplications and divisions.

An attempt is even made to introduce the Compatible Numbers Strategy in addition problems as students learn how to group addition numbers properly, in order to mentally calculate the result. Furthermore, flexible multiplications and divisions within the 100th are taught, using strategies such as the relationship between multiplication and division (for example, when a student knows that $8 \times 5 = 40$, they can conclude that $40 \div 5 = 8$) (CCSSM, 2011).

Lastly, an introduction is made to the Front - end Strategy in addition problems with whole numbers consisting of up to four digits. In Front - end Strategy, e.g. in calculating the sum of 442 + 236 + 378, students proceed as follows: 400 + 200 + 300 = 900 and 40 + 40 + 80 = 160, so 900 + 160 = 1060. (Reys, 1986).

3. Other surveys

The findings of surveys regarding the knowledge of in-service teachers' or prospective teachers' about estimation, vary. Educational interventions that have been conducted in the field of estimation and concern in-service teachers (Mildenhall, Hackling & Swan, 2009) or prospective teachers (Bestgen, Reys, Rybolt & Wyatt, 1980) are limited, and all of them focus on one type of estimation: computational estimation. Some of these studies evaluate teachers' abilities in estimation, without making any intervention in them (Dowker, 1992; Alajmi, 2009; Mindenhall et al., 2009; Tsao & Pan, 2013; Lemonidis & Kaimakami, 2013; Lemonidis, Mouratoglou & Pnevmatikos, 2014). Regarding the evaluation of this ability, some studies conclude that teachers are able to explain the importance of estimation, and to even use it to solve problems (Tsao & Pan, 2011). Other surveys find that the majority of teachers, approach computational estimation using rounding, while 5% of them do not know anything about estimation (Alajmi, 2009). There are even some teachers who believe that estimation works only as a "control device" (Mindenhall et al., 2009). Still, many of them do not execute estimations, instead they try to carry out the exact calculation (Lemonidis & Kaimakami, 2013).

However, there are some surveys that have studied the effects a professional learning intervention could have on teacher estimation abilities and have come up with positive results (Bestgen et al., 1980; Mildenhall, 2009; Mildenhall, Hackling, & Swan, 2009; Mindenhall & Hackling, 2012). However, each researcher's intentions are different, and therefore the professional learning interventions are formulated accordingly. Some surveys, through the professional learning intervention, aim at improving estimation ability, mainly its procedural aspect, i.e. at acquiring the knowledge of steps which should be followed in order to obtain the desired estimation result (Mindenhall & Hackling, 2012), while others seek to improve teachers' attitudes towards computational estimation (Bestgen et al., 1980; Mildenhall, 2009).

More specifically, the main aim of the majority of research interventions, as already mentioned, is to improve teacher estimation ability (Bestgen et al., 1980; Mildenhall, 2009; Mildenhall et al., 2009; Mindenhall & Hackling, 2012;), as well as to search for techniques that will work effectively in solving computational estimation problems (Bestgen et al., 1980). An effort has also been made to improve teacher pedagogical content knowledge on computational estimation (Mildenhall, 2009), and study their views on estimation (Mildenhall, 2009) and their feelings concerning it (Bestgen et al., 1980).

Professional learning interventions that have been implemented with teachers had the characteristics of action research or, in some cases, of case studies (Mildenhall et al., 2009). Each followed its own method according to researchers’ aims.

Lastly, surveys concerning students have also been carried out. These surveys examine how a professional learning intervention affects teachers and, by extension, their students (Mindenhall & Hackling, 2012), while there have also been educational interventions concerning students alone (Bobis, 1991; Star et al., 2009; Rittle-Johnson & Star, 2009; Lan, Sung, Tan, Lin & Chang, 2010). These studies are mainly aimed at learning estimation strategies (Bobis, 1991), as well as gaining flexibility in using these techniques and strategies (Star et al., 2009). There are also researchers who have dealt with students’ conceptual understanding concerning computational estimation (Star et al., 2009).

According to these surveys, students completely ignore estimation (Liu & Neber, 2012) or have moderate performance, while they perform better with natural numbers (Tsao & Pan, 2011). Regarding estimation strategies, results showed that the most commonly used estimation strategy for the students of all ages was the rounding to the nearest ten (Lemaire & Lecacheur, 2002).

This research proposes, for the first time, a professional learning intervention that concerns all types of estimation, that is based on a particular learning and teaching trajectory, and that studies the performance of both teachers and their students. Previous research has indicated that the knowledge of both primary and secondary school teachers, their pedagogical ability, and the progress of their students can be improved through participation in a continuous learning intervention that focuses on the development of mathematical thinking (Brendefur, Thiede, Strother, Bunning, & Peck, 2013). Mildenhall’s (2009) survey also identifies how learning communities can be used to improve the teaching of estimation.

It is also worth mentioning that this educational intervention is based on points where there is concern surrounding estimation (Siegler & Booth, 2004). The reasons why estimation is not taught as well as the most common teacher errors regarding estimation have also been taken into account in order to improve teaching (Ashlock, 2006). In addition, teacher preference on training methods (OECD, 2009) and factors contributing to effective teacher education have also been taken into consideration (Bobis, 1991). Moreover, learning and teaching trajectories of estimation (Anestakis & Lemonidis, 2014) were used, as were teacher training through learning communities (Mindenhall, 2009). Lastly, the gap in research literature concerning the training of estimation, was also noted as detailed above.

4. Methodology

The focus of teachers’ professional development worldwide is the application of action research through which teachers can evaluate their own practices and have the opportunity for self-improvement. Teachers’ professional development is implemented through the transition from traditional and dominant "training" methods, to modern action research (Kim, 2005). Therefore, this survey is an action research that concerns small-scale intervention in real-world operations, in this case a classroom, and a detailed examination of the intervention’s effects.

4.1. Purpose and research questions

The main purpose of this paper is to analyze the change of teachers’ performance after their participation in a professional learning intervention on estimations. The specific questions of this paper are as follows:

- What are the content knowledge (CK), the pedagogical content knowledge (PCK), and the knowledge of estimation strategies of teachers regarding estimation before and after intervention?

- Has there been a change in student knowledge after being taught by teachers involved in the intervention?
- Is there any relationship between teacher performance and student performance in estimation?

4.2. Participants

Participating teachers in this survey belonged to primary education and served in school units of a provincial Greek city (in urban and rural areas of the city). The learning community created consisted of 11 in-service teachers teaching in the 3rd Grade. This grade was chosen as it could lay the foundation of estimation, since nothing similar has been attempted before. The participants were selected by simple random sampling.

Apart from the teachers, all 105 students also participated in the survey. Students worked as a control device, since they were the recipients of teachers' teaching.

4.3. Research tools

Both qualitative and quantitative research tools were used. The research tools were: (a) questionnaire (both at the beginning and the end of the survey), (b) semi-structured interviews (at the end of the survey), (c) observation (in the classroom), and (d) reflection diary filled by every teacher and researcher too.

Data collection method triangulation was used to study the complexity of teachers' behavior in a more complete way, by looking at it from multiple angles.

4.4. Research procedure – Professional learning intervention plan

During the first stage of the survey students and teachers responded to a pre-test on the basis of which the initial knowledge they had on estimation was determined. The teacher pre-test included questions of content knowledge, while also aiming at exploring their pedagogical content knowledge.

The second stage of the survey constitutes the basic educational program, and consisted of eight (8) 120-minute courses. The total duration of the training program (6 training sessions, 2 follow-up tutorials, 10 future classroom visits/ per teachers' classroom) was 1 teaching year. The design, construction and implementation of the training sessions for teachers were based on the principles of both guided inquiry and discovery. Throughout the courses teachers were educated on the necessity of teaching estimation, on types of estimation, on estimation strategies, they practiced planning appropriate activities for the teaching of each strategy, and they learned how to manage misunderstandings regarding estimation. These courses combine various modes of training (presentation, group collaboration, combining theory and practice, activity planning, problem discussion, etc.) (Keneddy, 1998), which were in line with the preferences of the teachers. For the needs of the training, teachers acted within the framework a learning community, with a common vision, and serving a common goal: improving their knowledge and their teaching practices regarding estimation.

More specifically, during the first course a general discussion was carried out on estimation, its problems related to educational practice and teaching of estimation (group interview). The goals of the intervention were set by the researcher, and the plan of courses was handed out to teachers. The term of estimation was clarified and the frequency of use in daily life was pointed out by using examples cited by the researcher and teachers themselves. Reference was also made to the four types of estimation and examples were given for each type.

Throughout the second course teachers dealt with three types of estimation (number line estimation, measurement estimation, numerosity estimation). The teachers had the opportunity to identify the basic characteristics of each type of estimation, as well as plan related activities. They had the opportunity to gain the prerequisite knowledge of the estimation types as well as the strategies to implement them.

During the third and fourth course teachers dealt with the fourth type of estimation, that of computational estimation. During these courses teachers "were taught" various computational estimation strategies: front end strategy, rounding, accumulation, special numbers, compatible numbers, all of which had to be discovered by the teachers through examples. Throughout these activities teachers had to observe and note the characteristics of each strategy, as well as the circumstances that were most appropriate for each strategy.

The fifth course was a repetition of what teachers had already been taught. During the course, teachers had the opportunity to manage and separate all estimation strategies. They had the opportunity to compare strategies with each other, identify advantages and disadvantages of each strategy, as well as identifying the most appropriate circumstances for using each strategy.

The third stage of the survey included how teachers taught in their own classrooms over the course of an entire school year. At this stage the teachers received the estimation teaching and learning trajectory for 3rd graders, with indicative activities for each target that their students should achieve (a product of this study). Teachers tried to use this trajectory and material in math lessons. During this stage, teachers were observed teaching by the researcher (110 teaching hours), in order to detect changes in their teaching practices.

A trimester later, the sixth feedback course was held. Throughout the course, teachers had the opportunity to talk about their own classroom experiences, show how they approach estimation through textbooks, suggest activities, and reflect on the aforementioned. At the end of the second trimester, the seventh course was held, which was a feedback course as well. Its aims and proceedings were similar to that of the sixth meeting.

The program was completed at the end of the school year, when the final (eighth) course took place. The purpose of this course was to reflect on the overall program. Teachers had the opportunity to express their own opinion on the intervention's outcome i.e. whether the students benefited or not, but also weather the teachers themselves benefited from participating in the training process of estimation.

The fourth and final stage of the survey concerned the final evaluation of pupils and teachers. At this stage, teachers responded to a post-test with questions parallel to those they answered at the beginning of the survey, in order to identify changes in their content knowledge and pedagogical content knowledge. Students also completed a post-test similar to the one they initially answered, adapted to their knowledge after finishing the 3rd Grade. The main aim was to conclude if there was a change in their performance regarding the estimation, if they were influenced by the training process, and what factors influenced them.

4.5. Reliability and Validity of the survey

To ensure the reliability and validity of the research, a combination of research tools were used to test the same variables as in the research (questionnaire, interview, observation, reflection diary). For the sake of reliability, both questionnaires (of teachers and students) were piloted before conducting the research.

Clear instructions were given to participants on how to complete the questionnaires. The number of questions was satisfactory and the questions were clearly worded. The questionnaires were completed with the researcher being present.

However, some limitations emerged, because the participants were selected by simple random sampling. Such limitations were (a) some teachers serving in urban and others in rural areas, and (b) the number of pupils per class that varied, as there were both small numbers of pupils pre class (e.g. 2 pupils) as well as large numbers of pupils per class (e.g. 18 pupils).

4.6. Data analysis

The data collected from the survey were analyzed using the statistical software IBM SPSS Statistics 23, which was selected to draw conclusions on the quantitative data of research.

5. Results

Teachers participating in the survey were evaluated based on their performance in six areas: 1. The ability to formulate the definition of estimation, 2. content knowledge, 3. pedagogical content knowledge based on the findings of the questionnaires, 4. pedagogical content knowledge based on the observation of the teaching process, 5. estimation strategy use and 6. student performance in estimation problems. A comparison between performance in the pre-tests and post-tests revealed which evaluation areas presented statistically significant differences in teacher performance.

The following table (1) shows the existence or lack of statistically significant difference in teacher performance per evaluation area and overall:

Average teachers' performance per evaluation area							
Area	**N**	**Pre-test Mean**	**N**	**Post-test Mean**	**Wilcoxon**		
					Z	**P**	**St.Significance**
Content Knowledge	11	4.91	11	7.00	-2.971	0.003	S.S.D.*
Pedagogical Content Knowledge	11	6.55	11	8.00	-2.156	0.031	S.S.D.
Strategy Use	11	8.55	11	16.27	-2.944	0.003	S.S.D.
Total	**11**	**21.09**	**11**	**32.55**	**-2.937**	**0.003**	**S.S.D.**

*Statistically significant difference

** Statistically non-significant difference

Table1: Average teachers' performance per evaluation area in pre-test and post-test

The comparison between average teacher performance in the pre-tests and post-tests revealed that in all evaluation areas there is a statistically significant difference in their performance, apart from the area of "Estimation definition".

As far as overall teacher performance is concerned, statistically significant difference in their performance was identified, according to Wilcoxon test (z = -2.937, p = 0.003).

The same comparison was also sought in student performance in the pre-test and the post-test.

The table (2) below shows the existence or lack of statistically significant difference in student performance per student group and overall:

Average student performance per group							
Group	N	Pre-test Mean	N	Post-test Mean	Wilcoxon		
					Z	P	St.Significance
C1	20	2.95	20	3.60	-2.080	0.038	S.S.D.*
C2	16	3.31	16	5.44	-2.491	0.013	S.S.D.
C3	16	1.88	16	3.13	-2.324	0.020	S.S.D.
C4	13	4.85	13	6.46	-2.291	0.022	S.S.D.
C5	12	3.83	12	5.33	-2.141	0.032	S.S.D.
C6	10	2.00	10	2.80	-1.382	0.167	S.N.S.D.**
C7	5	4.60	5	4.60	0.000	1.000	S.N.S.D.
C8	5	3.80	5	6.20	1.841	0.066	S.N.S.D.
C9	3	4.00	3	5.33	-0.447	0.655	S.N.S.D.
C10	3	5.00	3	7.00	-1.604	0.109	S.N.S.D.
C11	2	5.50	2	6.00	-1.000	0.317	S.N.S.D.
Total	**105**	**3.34**	**105**	**4.65**	**-5.752**	**0.000**	**S.S.D.**

*Statistically significant difference

** Statistically non-significant difference

Table 2: Average student performance per group in the pre-test and post-test

By comparing the average student performance of 11 groups, it was observed that 5 of them (C1, C2, C3, C4, C5) showed a statistically significant difference in their performance.

Turning now to the set of 105 students, it was ascertained that the sample showed a statistically significant difference in their performance in estimation problems in the pre-test and post-test, according to the Wilcoxon test (z = -5.752, p = 0.000).

Below is the analysis of each evaluation area.

5.1. Content knowledge

Survey subjects were evaluated regarding content knowledge through their performance in 8 estimation problems, 5 of which were related to computational estimation, 1 to measurement estimation, 1 to numerosity estimation, and 1 to number line estimation (with a maximum score of 8). This evaluation was carried out both in the pre-test and post-test. From the combination of these two scores a final score was obtained for each teacher, according to which each was ranked in relation to the other teachers. Teachers were ranked according to their scores in the pre-test and post-test. Furthermore, the variation among scores in the pre-test and post-test were used for teachers ranking. If score variations of any teachers were the same, the teacher who was performing better from the beginning of the program was ranked higher.

The table (3) below shows the scores and teacher rankings regarding 'Content knowledge'.

Teacher ranking regarding 'Content knowledge'												
Teachers	**T1**	**T2**	**T3**	**T4**	**T5**	**T6**	**T7**	**T8**	**T9**	**T10**	**T11**	**Total**
Pre-test	5/8	6/8	4/8	5/8	3/8	6/8	6/8	4/8	5/8	3/8	7/8	54/88
Post-test	8/8	7/8	7/8	6/8	8/8	7/8	7/8	6/8	8/8	5/8	8/8	77/88
Total	13/16	13/16	11/16	11/16	11/16	13/16	13/16	10/16	13/16	8/16	15/16	-
Variation	+3	+1	+3	+1	+5	+1	+1	+2	+3	+2	+1	+23
Ranking	2nd	3rd	5th	4th	6th	3rd	3rd	7th	2nd	8th	1st	-

Table 3: Teacher ranking regarding 'Content knowledge'

In this case the change in teacher content knowledge, as shown by the change in their pre-test and post-test performance, is significant (Wilcoxon test, N = 11, z = 2.971, p = 0.003). Therefore, teacher content knowledge has significantly improved since they participated in the professional learning intervention on estimation.

5.2. Pedagogical content knowledge based on questionnaire findings

The subjects of the survey were evaluated regarding pedagogical content knowledge with a pre-test & post-test questionnaire, based on the sum of their total scores in three separate areas: (i) the introduction to estimation (1 question), (ii) the mode by which estimation was integrated into the curriculum (1 question), and (iii) the identification of student errors in the implementation of estimation (3 questions). This evaluation concerned the theoretical placement of teachers on the specific issues and was carried out through their responses to survey questionnaires.

More specifically, the score of the first sector (i) was based on the research of Desli & Anestakis (2014) who suggested four ways of introducing estimation, from the most effective to the least effective (numerical activity within context, logic crisis problems within context, problems relating to the size of numbers within context, and activities with benchmarks out of context) (with a maximum score of 4). The rating of the second area (ii) was based on Reys' (1986) theory on how to integrate estimation into the curriculum, suggesting three ways to integrate, from the most appropriate to the least appropriate (inclusion as an independent subject, inclusion at the end of each chapter, inclusion in relation to all areas of the curriculum) (with a maximum score of 3). Lastly, the third sector was evaluated through three questions related to identifying student errors in the implementation of estimation (with a maximum score of 3). This evaluation was carried out both in the pre-test and post-test. From the combination of these two scores a final score was obtained for each teacher, according to which each was ranked in relation to the other teachers. Ranking was carried out the same way as described above.

The table (4) below shows the scores and teacher rankings in the area of 'Pedagogical Content Knowledge based on questionnaire findings'.

Teacher ranking in the area of 'Pedagogical Content Knowledge based on questionnaire findings'												
Teachers	**T1**	**T2**	**T3**	**T4**	**T5**	**T6**	**T7**	**T8**	**T9**	**T10**	**T11**	**Total**
Pre-test	6/10	5/10	8/10	8/10	10/10	6/10	5/10	6/10	8/10	6/10	4/10	72/110
Post-test	10/10	6/10	9/10	10/10	9/10	9/10	7/10	7/10	6/10	9/10	6/10	88/110
Total	16/20	11/20	17/20	18/20	19/20	15/20	12/20	13/20	14/20	15/20	10/20	-
Variation	+4	+1	+1	+2	-1	+3	+2	+1	-2	+3	+2	+16
Ranking	4th	10th	3rd	2nd	1st	5th	9th	8th	7th	6th	11th	-

Table 4: Teacher ranking in the area of 'Pedagogical Content Knowledge based on questionnaire findings'

In this case, the variation of teachers' PCK (based on questionnaires) was significant (Wilcoxon test, N = 11, z = -2.156, p = 0.031). It was ascertained that the pedagogical content knowledge based on questionnaire data was improved after teacher participation in the professional learning intervention on estimations.

5.3. Pedagogical content knowledge based on the observation of the teaching process

In the area of pedagogical content knowledge based on teachers' teaching methods, subjects were evaluated through their total scores in five sub-sectors: (i) the integration frequency of estimation during the teaching of mathematics, (ii) the way in which estimation was included in the curriculum, (iii) the teaching method chosen, (iv) the ability to plan activities, and (v) the use of technology. This evaluation concerned how teachers taught these specific issues and was carried out through on-site researcher observation in teacher classrooms (a total of 110 observation sessions).

Regarding the first sector, the minimum integration number was the number of researcher on-site observations in each class (specifically, she carried out 10 visits per class), while the maximum integration number was integration in 15 lessons or more (in cases where number of lessons was over 10, the researcher collected the data by analyzing the reflection diaries). Accordingly, the minimum score a teacher could obtain was 1 (less than 10 lessons), while the maximum was 3 (more than 15 lessons). Regarding the inclusion of estimation in the curriculum, the score rating was based on Reys' (1986) theory described earlier. The third section was scored according to the effectiveness of learning theories, as suggested by international literature (Hargreaves, 1982) (behaviorism, constructivism, learning discovery) (with a maximum score of 3). The fourth sector was evaluated through the teachers' ability to plan estimation activities for their students, using a context, actions, and numbers that suit the strategy they wanted to teach (with a maximum score of 2). Finally, as far as the fifth sector is concerned, zero use was the minimum use of technology, while the use of technology 5 times or more was considered the maximum (i.e. more than half the times the researcher visited the classes) (with a maximum score of 3).

The table (5) below shows the scores and teacher rankings in the area of 'Pedagogical Content Knowledge based on the observation of the teaching process'.

Teacher ranking in the area of 'Pedagogical content knowledge based on the observation of the teaching process'											
Teachers	**T1**	**T2**	**T3**	**T4**	**T5**	**T6**	**T7**	**T8**	**T9**	**T10**	**T11**
Observation score	7/15	8/15	4/15	10/15	9/15	7/15	9/15	11/15	10/15	11/15	12/15
Ranking	6th	5th	7th	3rd	4th	6th	4th	2nd	3rd	2nd	1st

Table 5: Teacher ranking in the area of 'Pedagogical content knowledge based on the observation of the teaching process'

It is worth noting that in this area, teachers seem to have achieved good performances as regards their teaching quality.

5.4. Strategy use

The area of 'Strategy use' was evaluated through the combination of the scores of two sub-sectors, one relating to the number of strategies used by each teacher in two tests (the pre-test and the post-test) and the other relating to teacher knowledge of estimation strategies, according to the hierarchical classification level that strategies belonged.

For the first sector's evaluation, teachers responded to 8 estimation problems, 5 of which required computational estimation strategies, 2 required perceptual estimation strategies, and 1 required number line estimation strategies (with a maximum score of 8). Both the pre-test questions and post-test questions were evaluated. For the second sector, teacher evaluation was based on the findings of the Lemonidis & Likidis research (in press), which ranked computational estimation strategies on five levels through a hierarchical classification of them. Similar reasoning was also used to rank perceptual estimation strategies (Sarama & Clements, 2009), but also the number line estimation strategies (Peeters, Sekeris, Verschaffel & Luwel, 2017) (with a maximum score of 14). This evaluation was carried out both in the pre-test questions and the post-test questions.

From the combination of these two scores, a final score was obtained for each teacher according to which each was ranked as compared to the other teachers. Ranking was carried out the same way as described above.

The table (6) below shows the scores and teacher rankings in the area of 'Strategy use'.

Teacher ranking in the area of 'Strategy use'												
Teachers	**T1**	**T2**	**T3**	**T4**	**T5**	**T6**	**T7**	**T8**	**T9**	**T10**	**T11**	**Total**
Pre-test	7/22	14/22	8/22	8/22	7/22	11/22	11/22	6/22	6/22	5/22	11/22	94/242
Post-test	17/22	16/22	15/22	13/22	17/22	19/22	16/22	16/22	19/22	12/22	19/22	179/242
Total	24/44	30/44	23/44	21/44	24/44	30/44	27/44	22/44	25/44	17/44	30/44	-
Variation	+10	+2	+7	+5	+10	+8	+5	+10	+11	+7	+8	+85
Ranking	5th	1st	6th	8th	5th	2nd	3rd	7th	4th	9th	2nd	-

Table 6: Teacher ranking in the area of 'Strategy use'

In this case the change in teacher performance in the area of 'Strategy use' was significant (Wilcoxon test, $N = 11$, $z = -2.944$, $p = 0.003$). It is clear that after their participation in the training program teachers almost doubled their performance regarding knowledge and use of estimation strategies.

5.5. Student performance

For a better analysis of the results, the average performance score of each student group in two tests, one at the beginning and one at the end of the research, was calculated.

The table (7) below shows the average scores of student groups in the pre-test, post-test, and the final score for each group. According to these scores there was an overall ranking of all groups that took part in the survey. Ranking was carried out the same way as described above.

Teacher ranking in the area of 'Student performance'												
Teachers	**T1**	**T2**	**T3**	**T4**	**T5**	**T6**	**T7**	**T8**	**T9**	**T10**	**T11**	**Total**
Pre-test	2.95/9	3.33/9	1.88/9	4.85/9	3.83/9	2/9	4.60/9	3.80/9	4/9	5/9	5.50/9	41.47/99
Post-test	3.60/9	5.44/9	3.13/9	6.46/9	5.33/9	2.80/9	4.60/9	6.20/9	5.33/9	7/9	6/9	55.89/99
Total	6.55/18	8.77/18	5.01/18	11.31/18	9.16/18	4.80/18	9.20/18	10.00/18	9.33/18	12/18	11.5/18	-
Variation	+0.65	+2.13	+1.25	+1.61	+1.50	+0.80	0	+2.40	+1.33	+2.00	+0.50	+14.42
Ranking	9th	8th	10th	3rd	7th	11th	6th	4th	5th	1st	2nd	-

Table 7: Teacher ranking in the area of 'Student performance'

The change of student attitude towards estimation was positive according to their performance in the pre-test and post-test. This indicates a positive effect from the teaching they received from their teachers. This conclusion is also confirmed by the Wilcoxon test (N = 105, z = -5.752, p = 0.000). It is worth mentioning that this teaching was based on a specific learning and teaching trajectory, a fact that indirectly indicates the effectiveness of the trajectory.

5.6. Correlation of teacher evaluation areas

The following table (8) shows teacher ranking in each of the six evaluation areas and, lastly, the overall ranking of teachers based on all areas together.

Teachers	**CK**	**Strategy use**	**PCK (questionnaires)**	**PCK (teaching process)**	**Student performance**	**Total**	**Ranking**
T1	2nd	5th	4th	6th	9th	26	5rd
T2	3rd	1st	10th	5th	8th	27	6th
T3	5th	6th	3rd	7th	10th	31	8th
T4	4th	8th	2nd	3rd	3rd	20	2nd
T5	6th	5th	1st	4th	7th	23	3rd
T6	3rd	2nd	5th	6th	11th	26	5th
T7	3rd	3rd	9th	4th	6th	25	4th
T8	7th	7th	8th	2nd	4th	28	7th
T9	8th	4th	7th	3rd	5th	27	6th
T10	8th	9th	6th	2nd	1st	26	5th
T11	1st	2nd	11th	1st	2nd	17	1st

Table 8: Overall teacher ranking in all areas

In order to determine whether there is some kind of correlation between teacher evaluation areas, the Kendall W indicator was used, as shown in the table (9) below. This indicator was used to evaluate the degree of agreement among evaluation areas in the ranking of teachers.

Teacher evaluation areas correlations							
N=11			**CK**	**PCK (questionnaires)**	**Strategy use**	**Student performances**	**PCK (teaching process)**
Kendall W indicator	**CK**	Correlation	1.000	-.472*	**.495***	-.208	-.137
		Sig. (2-tailed)	.	.048	.039	.385	.576
	PCK (questionnaires)	Correlation	-.472*	1.000	-.440	-.164	-.321
		Sig. (2-tailed)	.048	.	.061	.484	.180
	Strategy use	Correlation	**.495***	-.440	1.000	.000	**.038**
		Sig. (2-tailed)	.039	.061	.	1.000	.874
	Student performances	Correlation	-.208	-.164	.000	1.000	**.850**
		Sig. (2-tailed)	.385	.484	1.000	.	.000
	PCK (teaching process)	Correlation	-.137	-.321	**.038**	**.850**	1.000
		Sig. (2-tailed)	.576	.180	.874	.000	.

Table 9: Teacher evaluation areas correlations

The ‘**Content Knowledge’** area has a negative correlation with all evaluation areas, as the W values in three out of four areas are -0.472, -0.208, and -0.137. This area and its relation to the other areas seems to be influenced by factors such as the teachers' interest in understanding concepts, the teachers’ ages and years of service, as well as the teachers’ ability to assimilate and harmonize the new proposed trajectory with the existing curriculum, as mentioned earlier. The only exception is the ‘Strategy use’ area, which shows moderate positive correlation (W = 0.495), as 0.3 <W <0.65. This may be due to factors such as the teachers’ relationship with mathematics, which arose from the separate analysis of each area.

The ‘**Pedagogical Content Knowledge’** area based on the questionnaire examination presents a negative correlation with all the other evaluation areas, as the W values of areas are -0.472, -0.440, -0.164, and -0.321.

The ‘**Strategy use’** area seems to not affect the majority of evaluation areas. More specifically, there is a weak positive correlation with ‘PCK according to teaching process’ (W = 0.038) areas, as 0 <W <0.3, a moderately positive correlation with the area ‘Content knowledge’ (W = 0.495), as 0.3 <W <0.65, while there seems to be no correlation with the area of ‘Student Performance’ (W = 0.000).

The ‘**Student Performance’** area has absolutely no correlation to ‘Content Knowledge’, ‘Strategy Use’ and ‘Pedagogical content knowledge based on the questionnaire’ areas. However, it appears to have a strong positive correlation with the pedagogical content knowledge based on teaching practice (W = 0.850), as 0.65 <W <1.

Lastly, ‘**Pedagogical Content Knowledge’** based on teaching practice seems to have a weak positive correlation with the ‘Strategy use’ area (W = 0.038), as 0 <W <0.3, a strong positive correlation with the ‘Student Performance’ area (W = 0.850) as previously reported, while not correlated to ‘Pedagogical content knowledge based on the questionnaire’ (W = -0.321).

It is generally understood that:

- Pedagogical content knowledge based on questionnaire examination and pedagogical content knowledge based on teaching practice are not related to each other. This fact suggests that questionnaire data alone is not enough to draw conclusions on PCK, and that a more complete and truer picture is provided by teaching observation.
- Student performance is only influenced by the way in which classroom teaching takes place, i.e. it is influenced by the ‘pedagogical content knowledge based on the teaching process’.
- Content Knowledge only impacts the area of Strategy use.

The analysis carried out in this study has allowed for an overall view of educational profiles, as there has been a combination of separate evaluation areas for this purpose. Indicative examples of two educational profiles:

Teacher T11 was first in the overall ranking of teachers, had a very good content knowledge of estimation (1st), and of estimation strategy use (2nd), he also taught estimation in class very well (1st), although his answers regarding PCK questions on the questionnaire were wrong and ranked him last (11th). Furthermore, it was found that his students performed very well in estimation problems (2nd).

On the other hand, teachers T3 and T6 were last in the overall ranking of teachers. More specifically, teacher T3 had an average content knowledge of estimation (5th) and an average performance in strategy use (6th), while his teaching practice was below average (7th), although his responses regarding PCK questions on the questionnaire placed him third in the ranking. As a result, it was found that his students performed poorly on estimation problems (10th).

Also, teacher T6 had good content knowledge of estimation (3rd) and estimation strategy use (2nd), but his teaching practice was average (6th). His PCK was also average according to his replies on the questionnaire (5th). As a result, his students performed poorly in estimation problems (11th).

6. Conclusion

The results of the pre-test showed that the majority of teachers were not able to explain the importance of estimation, which came into contrast with prior research results of Tsao & Pan (2013). However, there were some teachers who were able to use estimation for solving problems, without knowing the variety of estimation strategies.

Surveys carried out regarding teachers’ pedagogical content knowledge in estimation are very limited (Mindenhall, 2009). In particular, they are limited to the description of the possibility of improving pedagogical content knowledge through participation in a training program, instead of describing the level and quality of that knowledge. The results of this study showed that teachers preferred more decontextualized ways of introducing estimation, prior to their participation in the training program.

Regarding the students, the research carried out shows the same range of results as with the teachers. More specifically, there are studies that conclude that students are completely unaware of computational estimation (Liu & Neber, 2012), while others claim that they have moderate performance in computational estimation, with better performance in natural numbers (Tsao & Pan, 2011). The students of this study belong to the first category, since their initial tests results indicated that they had no knowledge regarding estimation, as they recorded low or moderate performance in estimation problems (pre-test student mean score 3.34).

Therefore, according to the results of teachers' pedagogical content knowledge on estimation, the need of training teachers through a professional learning program in order to achieve better teacher training in estimation was made evident. According to literature on the matter, the knowledge of primary and secondary school teachers, their pedagogical ability and the progress of their students may be increased by participating in a continuous professional training program focused on the development of mathematical thinking (Brendefur et al., 2013).

The conclusion regarding the need of teacher training on estimation is also reinforced by the results of this study, as content knowledge, pedagogical content knowledge, the use of estimation strategies, and student performance, are improved after teachers participated in the organized training program that was carried out for the needs of this study. This is made obvious by the performance changes from the pre-test to the post-test (+23, +16, +85, +14.42) which is consistent with the literature (Mildenhall, 2009). In particular, most teachers were able to achieve higher scores in the final evaluation tests, using a variety of strategies with great success, a finding consistent with the Tsao & Pan survey (2013). As far as their pedagogical content knowledge is concerned, it seemed to improve, as post-training teachers turned to more experiential and playful ways of teaching estimation. They even revised their views and the majority of them realized that estimation could be included in almost all chapters of mathematics (Reys, 1986). Regarding student mistakes and their detection, teacher performance remained very good, the only difference being that after the training, they were able to identify the strategy used by students in each case.

However, it is worth mentioning that the results of PCK based on the findings of the questionnaires and the findings of observation of the teaching process are not related to each other. This survey's findings make it clear that the investigation of PCK's quality cannot be carried out exclusively through a questionnaire, as has been attempted by other surveys in the past (Mildenhall, 2009). Instead, it is important to combine questionnaire data and data from the observation of the teaching process, because this method produces the more sound conclusions.

As regards students, their performance improved when students received systematic instruction from their trained teachers, as shown by the change in their performance in the pre-test and the post-test (post-test student mean score 4.65). This change is statistically significant (Wilcoxon test, $z = -5.752$ and $p = 0.000$).

International literature refers to the teaching of estimation through learning trajectories and its advantages. The learning and teaching trajectory of estimation designed for the needs of this study for the 3rd Grade was quite effective, as students were able to improve their performance after the instruction they received from their teachers, which was based on the targets and proposed activities of that trajectory.

The only difficulty identified related to one of the computational estimation strategies, namely, the Front - end Strategy, which students used to a lesser extent than all estimation strategies. In general, students and teachers significantly improved in all types of estimation with the help of this particular trajectory. This conclusion was confirmed by Van den Heuvel-Panhuizen (2001), who states that teaching through learning and teaching trajectories ensure a smooth transition from the spontaneous application of strategies (informal phase) to the conscious application of rules, with a view to reaching the flexibility phase.

Finally, many researchers (Chinnappan, 2000; Green, Piel, & Flowers, 2008; Lin, 2011) have also dealt with the use of technology to train teachers in various mathematical sections. There is even reference to the fact that present technological tools can help to better understand mathematical subjects. This study, cannot confirm this to a great extent, as the majority of teachers, with one exception, did not make use of technological advantages during their teaching. The reason that prevented them from using technology in almost all cases was the lack of technological infrastructure, or other technical problems that made this type of teaching difficult. However, one teacher (T10) who utilized technology in almost all courses, achieved a better student understanding, since this student group (C10) was placed first in the group ranking. Naturally, one cannot use this example to generalize.

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