New Trends in Mathematics Teaching in Teacher and Methodologist Training for the Preprimary and Primary Education

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Abstract
The aim of this contribution is to present mathematics study plans for individual degrees of study branch Pre-school and elementary pedagogy. We refer to the changes which result from presented study plans and which bring brand new conception of education of mathematics in training of 1st degree primary school teachers. In the paper we present also the research that focused on issues of teaching quantifiers at elementary schools and statistical evaluation of this research.

Key words: trends in mathematics teaching in teacher, new conception of education, primary education, nonparametric methods of mathematical statistics

JEL classification: I210

1. Introduction
Accession of Slovakia to the European Union has brought great changes in education at our universities. In order to provide students with comparable education with the developed European universities, many universities in Slovakia offer students a variety of new courses and adapt curricula so that students can continue in their studies at any European university either in master or doctoral degree and also to allow students to have the opportunity to study in selected semesters at other universities within the exchange programs of universities. Faculty of Education of Constantine the Philosopher University in Nitra had the right to run the study course for primary education teacher training in the past on the basis of decision of the Accreditation Commission at the Ministry of Education of Slovak Republic. Accreditation of new programs and re-accreditation of existing study courses in 2005 completed a complex reconstruction of the organization of study courses of our university. Except other study
courses at the Faculty of Education, the study program of the field of study *Preschool and Elementary Pedagogy* was accredited. Within this study program, Faculty of Education in Nitra has changed the system of teacher training and has offered the possibility of Bachelor's degree for kindergarten teachers since school year 2005/2006. These changes were adopted in accordance with the unification of the teacher training at universities in the European Union and also in accordance with the current needs of the modern era. The presented article deals with changes resulting from the new concept of mathematics teaching in training teachers for the first degree of elementary schools.

2. Study plan of mathematics in study programs within the field of study Preschool and Elementary Pedagogy

At present, the new system of methodologist and teacher training courses for the field of pre-primary and primary education takes place within the field of study *Preschool and Elementary Pedagogy*. Under the system of study fields, students can study in this study field in three degrees:

- in the first (bachelor) degree of university education with a standard length of study 3 years,
- in the second (master) degree of university education with a standard length of study 2 years,
- in the third (doctoral) degree of university education with a standard length of 3 years.

For applicants for Master degree studies, who completed first-degree studies in another but related field, a master program of study with the length of study 3 years can be proposed. Graduates of this field of study are according to the level of obtained education qualified for the following professions:

- graduates of the first degree are competent as kindergarten teachers, educator in institutions for children in preschool and early school age (especially in school clubs and leisure time centers),
- graduates of the second degree are qualified to become the teachers for primary education, methodologist in the field of pre-primary and primary education, a specialized employee of the state administration in the field of education of children in preschool and school age,
- graduates of the third degree are competent for the profession of scientific researcher in the corresponding field of pedagogy.

As the staff of the Department of Mathematics of Faculty of Natural Sciences in Nitra provides ten subjects for students of this field at our university, the Department of Mathematics has developed the study plan of teaching mathematics the students of this field of study. Changes resulting from this study plan were substantial and brought a whole new concept in teaching mathematics within the preparation of teachers for the first degree of elementary schools. The concept of teaching mathematics within the field of study *Preschool and Elementary Pedagogy* is clearly displayed in the Table 1.

Parts of the curriculum syllabus of Mathematics in the field of study Preschool and Elementary Pedagogy are information sheets prepared by the guarantees of individual subjects. In order to introduce new approach to teaching mathematics within the presented training courses, we present the aims of individual subjects from information sheets.

*Course title: Basics of Mathematics and Development of Mathematical Logic*

*Aim of the subject:* Training of students should be focused on the analysis of tasks and activities with mathematical elements in the employments of children in kindergartens. Highlight the need for
mathematics in everyday life and create positive attitude towards mathematics in students by using suitable tasks. Provide students with basic knowledge about logic and intuitive set theory. Develop language and formal aspects of mathematical expression means. Develop logical reasoning ability and deepen their theoretical knowledge by solving practical problems. Those students who will continue in master studies Teacher Training for Primary Schools are obliged to choose the subjects marked with an asterisk *.

**Course title: Development of Mathematical Concept of Numbers**  
**Aim of the subject:** To gain basic knowledge about introduction of natural numbers, and deepen knowledge about records of natural numbers in decimal system, and about algorithms of calculating with natural numbers in decimal system. Acquisition of skills to solve mathematical problems of kindergarten, in which numbers and operations with numbers appear.

**Course title: Basics of Elementary Arithmetic**  
**Aim of the subject:** To gain basic knowledge about motivation of introducing basic numeric sets (sets \( \mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R} \)) and about their construction.

**Course title: Basics of Elementary Geometry**  
**Aim of the subject:** To gain basic knowledge about position and metric properties of planar and spatial objects. Master basic sets of points of a given property and the topic of identical geometric transformations. Elaboration of mathematical terminology and symbolism, with an emphasis on the process of creating concepts. Improvement of the aesthetics of graphic expression.

**Course title: Arithmetic - Selected Problems**  
**Aim of the subject:** To deepen and broaden knowledge about numbers and their records and properties. Record of natural numbers in other numeric systems. Counting in other numeric systems.

**Course title: Workshops in Mathematics**  
**Aim of the subject:** Development of the combinatorial thinking through solving practical problems and problems of real life.

**Course title: Geometry - Selected Problems**  
**Aim of the subject:** To familiarize students with geometric transformations and their usage within solving construction tasks.

**Course title: Workshops in Geometry**  
**Aim of the subject:** To gain basic knowledge and skills to solve geometric tasks. Developing skills to create geometric models.

**Course Title: Didactics of Mathematics 1**  
**Aim of the subject:** To familiarize students with contents, methods and forms of work in teaching mathematics on the 1st degree of elementary school. Prepare students for the continuous pedagogical practice. Lead students towards systematic and regular work with mathematical and didactic literature.

**Course Title: Didactics of Mathematics 2**  
**Aim of the subject:** To lead students towards the ability to solve tasks from the thematic units of Mathematics for the 1st degree of elementary schools together with students of the 1st degree of elementary school. Prepare students for pedagogical practice on the 1st degree of elementary schools.

**Course Title: Computer and Didactic Games**
Aim of the subject: Bring working of computers closer to the students so that it was understandable for students of the first degree of elementary school. Explain work with computer as a part of extracurricular and after-school activities. Teaching with the computer, its strengths and weaknesses. Search on the Internet.

Course Title: Methods of Solving Mathematical Problems
Aim of the subject: To familiarize students with methods of solving tasks from Arithmetic, Geometry and entertaining tasks.

Title of Course: Entertaining Tasks
Aim of the subject: To familiarize students with the possibilities of motivation within teaching mathematics by solving entertaining tasks and tasks with historical and fairytale motifs.

Course Title: Mathematical Competitions
Aim of the subject: To acquaint students with mathematical competitions at school. Prepare them for leadership of mathematical groups and for organizing mathematical competitions.

For successful implementation of the transformation of old curriculum into new curriculum it was necessary not only to create new curriculum, but also the creation of new textbooks for each of the mathematical disciplines of the field of study Preschool and Elementary Pedagogy.

3. Research focused on issues of teaching quantifiers in elementary schools

We present in this part the research that focused on issues of teaching quantifiers in elementary schools. Our objective was to determine how students are able to learn to work with quantified statements and negate them, and to what extent students are able to understand their symbolic expressions and apply the knowledge about the respective roles of quantifiers. We designed an experimental way of teaching the topic Quantified statements, which aims to make the teaching of this topic more effective. Results of the experiment were evaluated with nonparametric statistical methods. The results confirmed that the proposed teaching model is effective.

3.1. Quantifiers in the Subject Matter of Mathematics for the Primary School

Because one of very important parts of mathematics is mathematical logic, we were interested to what extent mathematical logic is reflected in the curriculum of Mathematics for elementary schools. The level of abstract thinking of children of younger school age is relatively low. In case that children think about the statement veracity, they are governed by their own experience, they consider the statement veracity on the basis of the particular situation. The aim of introducing elements of propositional logic into teaching children in preschool and early school age is that children should obtain the need to reason, to feel the importance of logic, especially when formulating strict rules, and that they should learn to use the language of logic on the propaedeutic level.

There are tasks and activities in the subject matter of Mathematics on the primary level in elementary school, which constitute the propadeutics of notions statement and truth value of a statement. The notion statement is replaced by notions expression, theorem, claim, and the like. Tasks are assigned in order to determine the veracity of a given statement.

As part of additional subject matters of the eighth grade of primary school pupils familiarize with statements, truth value of statement, negation of statement, and statements containing data of an amount.
According to the experience of teachers, students have problems with negation of certain statements, particularly of quantified statements, and they also have problems with proper formulation of mathematical statements. Therefore the authors of work Melichová, S., Tírpáková, A., Markechová, D. (2007) proposed an experimental method of teaching the topic Quantified Statements, which aimed to eliminate problems described above. We have developed model lessons for teaching of quantified statements. Proposed experimental teaching was included in teaching mathematics of students in the eighth year of study following the expanding subject matter Elementary Knowledge of Logic.

The aim of this experimental method of teaching was:
- to highlight the importance and utilization of mathematical logic and quantifiers not only in Mathematics, but also in everyday life, within communication either in spoken or in written form;
- to emphasize the influence of mathematical logic and quantifiers within development of mental operations;
- to contribute to the improvement of logical thinking of students through suitably selected tasks;
- to clarify the meaning of words all, every, some, at least one, which occur normally in everyday communication, however, when working with mathematical problems, they can cause problems;
- to teach students to use quantified statements and negate them properly;
- to teach students to use correct and precise expressions and to formulate ideas correctly;
- to point out the inaccuracies and ambiguities in expressions and related difficulties, problems and misunderstandings in everyday life;
- to teach how to prevent errors in reasoning and expressions;
- to compile a collection of logic tasks with emphasis on quantified statements, which would be useful both in teaching mathematical logic and also in other thematic units of Mathematics as motivation problems;
- to bring closer the history of logic as a motivational element of the teaching process.

3.2. Methodology and results of the research

The experiment was realized in school year 2006/2007 at two elementary schools in Trnava and Šaľa. A total of 96 students from the eighth grade classes participated in the experiment, 49 students of which were from Trnava and 47 pupils from Šaľa. We randomly divided students into four groups - two experimental and two control groups.

The first experimental group was made up of 25 pupils of eighth grade from elementary school in Trnava and the second experimental group of 22 pupils of eighth grade from elementary school in Šaľa. The first control group was made up of 24 pupils of eighth grade from elementary school in Trnava and the second control group of 25 pupils of eighth grade from elementary school in Šaľa. Before starting the experiment, all pupils in experimental and control groups were assigned a control test, whose aim was to determine whether the level of pupils' knowledge in control and experimental groups is the same. Statistically significant differences in the level of pupils' knowledge in experimental and control groups could significantly distort the outcome of the experiment.

The control test consisted of seven tasks for which it was possible to obtain a maximum of 31 points. Tasks were selected from second part of Mathematics textbooks for the eighth grades of elementary schools. The control test contained following subject matters of Mathematics for the 8th grades: Inequalities (6 points), set of points of given property (4 points); construction task aimed at
using set of points of a given property (7 points), functions (6 points), probability (4 points) and two simple logical tasks (2 + 2 points). The control test was at first evaluated in percentage (Table 2). The table 2 shows that there are not differences between the results of test of created groups, i.e. the level of students’ mathematical knowledge from control groups and experimental groups can be considered as equal.

Extension topic **Elementary Knowledge of Logic** was explained to all groups of pupils. After mastering this topic, pretest was assigned to pupils. It consisted of five tasks, in which students had to:

1. decide which of the given sentences are statements;
2. determine the truth value of the statements;
3. produce a negation of the statement;
4. find out which of the quantified statements are true and which are false;
5. produce a negation of the quantified statements.

It was possible to obtain a maximum of 17.5 points in the pre-test. Maximum achievable points for individual tasks in pre-test were as follows: 1st task: 1.5; 2nd task: 1.5; 3rd task: 4.5; 4th task: 3; 5th task: 7 points.

Table 3 shows the average points gained in the pre-test for individual tasks in experimental and control groups.

The average values of obtained points in pre-test in percentage were also visualized graphically (Figure 1).

While the average points gained in the first four tasks are relatively high when compared with the maximum point values, in the fifth task it is not true. It was exactly the fifth task which was focused on the negation of quantified statements. Thereby our experience was confirmed, i.e. majority of problems that students have are with those tasks, in which there are quantifiers.

Hypothesis about the equal knowledge level of students of experimental and control groups in results of pre-test in chosen schools was verified using Wilcoxon Two-Sample Assuming test. Wilcoxon Two-Sample Assuming test is one of the most widely used nonparametric methods of mathematical statistics. It is used as a nonparametric alternative to parametric t-test for two independent sample sets. The hypothesis under test is null hypothesis, which is formulated as follows:

\[ H_0 : \text{The samples come from the same base set, i.e. there is not statistically significant difference between the two groups of students with regard to the results obtained in the test.} \]

Null hypothesis will be tested against alternative hypothesis:

\[ H_1 : \text{The samples don’t come from the same base set, i.e. there is statistically significant difference between the two groups of students with regard to the results obtained in the test.} \]

The test was conducted using program STATISTICA. After entering the input data, in the report output of the computer we get these results for the selected Wilcoxon Two-Sample Assuming test: the value of the test criteria \( z \) and the value \( p \), which is the probability of error that we committed, if we reject the tested hypothesis.

If the calculated \( p \) value is sufficiently small (\( p < 0.05 \) respectively \( p < 0.01 \)), we reject the tested hypothesis (on the significance level 0.05 respectively 0.01). Otherwise we cannot reject the hypothesis, the observed differences are not statistically significant. After entering the results of pre-test, we got the following output report (Table 4).

Since the value of probability \( p \) is a large number, we cannot reject the tested hypothesis \( H_0 \). The results obtained using the Wilcoxon Two-Sample Assuming test confirmed that there was not
statistically significant difference in the level of knowledge of students in mathematics between the experimental and control group in both schools before the start of the experiment. Subsequently, we proceeded to the implementation of the experiment itself. In the experimental groups, we expanded the subject matter of mathematical logic by adding quantified statements. We taught students of the experimental groups to work with quantified statements and negate them in four hours seminar.

After completion of the experiment we gave the same test – posttest - to pupils of both groups (experimental and control groups). Posttest consisted of three tasks in which it was necessary:
- to decide which of these claims is true;
- to add quantifiers into sentences so that created statement was true;
- to negate quantified statements.

Results, which students achieved in posttest, showed that there was significant improvement in work with quantified statements of students in experimental groups in comparison with control groups of students. Table 5 shows the average values of points gained in each task of posttest. With the help of Wilcoxon Two-Sample Assuming test, realised in the program STATISTICA, we verified if there is statistically significant difference in the level of knowledge about quantifiers of the pupils of experimental and control groups. After entering the results of posttest we got the following output report (Table 6).

Since the value of probability $p$ is a small number, we reject the tested hypothesis $H_0$ on the level of significance $\alpha = 0.01$. The results obtained using the Wilcoxon two-sample test confirmed that there is statistically significant difference in the level of knowledge of students in mathematics between the experimental and control group in both schools after the realization of the experiment.

### 3.3. Discussion

Statistically, it was proved that the realized experiment has brought expected results, which resulted in significant improvement in pupils' work with quantifiers. Students learnt not only the proper use of quantified statements, but by the work with the statements they also revealed common imperfections and inaccuracies in their own expressing.

They also found out that mathematics extends into areas with which it has nothing in common at first sight and that mathematical logic is not only entertaining tasks and games. Through quantifiers and the negation of quantified statements, only a small part of complicated formalism, symbolism and a high level of abstraction, which mathematical logic conceals in itself, was uncovered for students.

### 4. Conclusions

Opening Bachelor's study of the field of study *Preschool and Elementary Pedagogy* has provided a whole new concept in training teachers for the first degree of primary schools. Because graduates of these studies should be competent to perform in professions kindergarten teacher and educator in institutions for children in preschool and early school age, teaching mathematics must be consistent with this objective. Mathematical training of students must be focused so that they learn to shape the elements of mathematical education for preschool children. This requires new methodological approaches on the part of the teachers at the Department of Mathematics. We tried to incorporate these methodological approaches into new textbooks and study materials.
References

[10] Information sheets of mathematical subjects of the study program of study branch Pre-school and elementary pedagogy, Department of Mathematics, Faculty of Natural Science, Constantine the Philosopher University in Nitra, 2004, http://www.ukf.sk/ (in Slovak)
### Tables and Figures

**Table 1. The concept of teaching mathematics within the field of study Preschool and Elementary Pedagogy**

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Table 2. Results of control test in percentage

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Table 3. Average values of obtained points in pre-test

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Table 4. The results of the Wilcoxon Two-Sample test

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Table 5. Average values of obtained points in posttest

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Table 6. The results of the Wilcoxon Two-Sample test

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Figure 1. Average values of obtained points in pre-test in percentage